1. (a) Express $\dfrac{3x}{(2x-1)(x-2)}$ in partial fraction form.

(3)

(b) Hence show that

$$\int_5^{25} \frac{3x}{(2x-1)(x-2)}\,dx = \ln k$$

where k is a fully simplified fraction to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

October 2022/Q2

2.
$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C

(4)

(b) (i) Hence find $\int f(x)\,dx$

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

(6)

3.

(a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.

(3)

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \dfrac{1}{2}$ when $y = \dfrac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.

(6)

June2022/Q2

4.
$$f(x) = \frac{5x+10}{(1-x)(2+3x)}$$

(a) Write $f(x)$ in partial fraction form.
(3)

(b) (i) Hence find, in ascending powers of x up to and including the terms in x^2, the binomial series expansion of $f(x)$. Give each coefficient as a simplified fraction.
(5)

(ii) Find the range of values of x for which this expansion is valid.
(1)

January 2023/Q1

5.
$$f(x) = \frac{8x-5}{(2x-1)(4x-3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.
(3)

(b) Hence find $\int f(x)\,dx$
(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$
(5)

June2023/Q3

6. (a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.
(3)

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \dfrac{1}{2}$ when $y = \dfrac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.
(6)

June2022/Q2

7. (a) Express $\dfrac{3x}{(2x-1)(x-2)}$ in partial fraction form.

(3)

(b) Hence show that

$$\int_5^{25} \frac{3x}{(2x-1)(x-2)}\,dx = \ln k$$

where k is a fully simplified fraction to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

October 2022/Q2

8. Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2}\,dx$$

giving your answer in the form $p\ln q + r$ where p, q and r are rational numbers.

(6)

January 2024/Q2

9. $$f(x) = \frac{8x-5}{(2x-1)(4x-3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

(5)

June2023/Q3

10. $$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants A, B, C and D.

(5)

A curve has equation

$$y = g(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

(2)

(c) Hence, explain why $g'(x) > 3$ for all values of x in the domain of g.

(1)

October 2021/Q3

ANSWERS:

2 (a)

$$\frac{3x}{(2x-1)(x-2)} \equiv \frac{A}{2x-1} + \frac{B}{x-2} \Rightarrow \text{Values for } A \text{ and } B$$

One correct value, either $A = -1$ or $B = 2$

Correct PF form $\frac{-1}{2x-1} + \frac{2}{x-2}$

(b)

$$\int \frac{-1}{2x-1} + \frac{2}{x-2}\,\mathrm{d}x = -\frac{1}{2}\ln(2x-1) + 2\ln(x-2) + (c)$$

$$\int_5^{25} \frac{3x}{(2x-1)(x-2)}\,\mathrm{d}x = \left[-\frac{1}{2}\ln(2x-1) + 2\ln(x-2)\right]_5^{25}$$

$$= \left(-\frac{1}{2}\ln 49 + 2\ln 23\right) - \left(-\frac{1}{2}\ln 9 + 2\ln 3\right)$$

$$\ln\frac{529}{21}$$

3(a)	$1 = A(3x-1)^2 + Bx(3x-1) + Cx$
	$x \to 0 \quad (1 = A)$
	$x \to \frac{1}{3} \quad 1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct coefficients of x^2
	$0 = 9A + 3B \Rightarrow B = -3$ all three constants correct
(b)(i)	$\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$ $= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$ $\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$
(b)(ii)	$\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$
	$= \ln \frac{2 \times 2}{5} + \ \dots$
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$

2(a)	$\left(\frac{1}{(1+3x)(1-x)} = \frac{A}{1+3x} + \frac{B}{1-x} \Rightarrow \right) 1 = A(1-x) + B(1+3x)$
	when $x = 1 \Rightarrow 1 = 4B \Rightarrow B = \dots$ or when $x = -\frac{1}{3} \Rightarrow 1 = \frac{4}{3}A \Rightarrow A = \dots$
	$\frac{3}{4(1+3x)} + \frac{1}{4(1-x)}$
(b)	$\int \cot y \, dy = \int \dots dx \Rightarrow \text{"}\ln \sin y\text{"} = \int \dots dx$
	$\dots = \int \left(\text{"}\frac{3}{4(1+3x)}\text{"} + \text{"}\frac{1}{4(1-x)}\text{"} \right) dx = \dots \ln(1+3x) \pm \dots \ln(1-x) \quad (+c)$
	$\ln \sin y = \frac{1}{4}\ln(1+3x) - \frac{1}{4}\ln(1-x) \quad (+c)$ oe
	$\ln \sin\left(\frac{\pi}{2}\right) = \frac{1}{4}\ln\left(1 + 3 \times \frac{1}{2}\right) - \frac{1}{4}\ln\left(1 - \frac{1}{2}\right) + c \Rightarrow c = \dots \left(= -\frac{1}{4}\ln 5 \right)$
	$k \ln \sin y = m \ln(\dots) \Rightarrow \sin^k y = \dots^m$ or $k \ln \sin y = \dots \Rightarrow \sin^k y = \exp(\dots)$
	$\sin^4 y = \frac{1+3x}{5(1-x)}$

1 (a)	$\frac{5x+10}{(1-x)(2+3x)} \equiv \frac{A}{1-x}+\frac{B}{2+3x} \Rightarrow$ Value for A or B
	One correct value, either $A=3$ or $B=4$
	Correct PF form $\frac{3}{1-x}+\frac{4}{2+3x}$
(b)(i)	$\frac{A}{1-x}=A(1-x)^{-1}=A\left(1+x+x^2+\ldots\right)$
	$\left\{\frac{B}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1}=\left\{\frac{B}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\ldots\right);=\frac{B}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)$
	$f(x)=\ 3\times\left(1+x+x^2+\ldots\right)+\ \frac{4}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)$
	$=5+\frac{15}{2}x^2+\ldots$
(b)(ii)	$\lvert x\rvert<\frac{2}{3}$

3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1}+\frac{B}{4x-3} \Rightarrow 8x-5=A(4x-3)+B(2x-1) \Rightarrow A=\ldots\ B=\ldots$
	$A=1$ or $B=2$
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1}+\frac{2}{4x-3}$
(b)	$\int\left(\frac{1}{2x-1}+\frac{2}{4x-3}\right)dx=\frac{1}{2}\ln\lvert 2x-1\rvert+\frac{1}{2}\ln\lvert 4x-3\rvert(+c)$ Follow through their A and B
(c)	$\left[\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(4x-3)\right]_k^{3k}=\frac{1}{2}\ln(6k-1)(12k-3)-\frac{1}{2}\ln(2k-1)(4k-3)$ $=\frac{1}{2}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$
	$\frac{1}{2}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=\frac{1}{2}\ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=20$ $\Rightarrow(6k-1)(12k-3)=20(2k-1)(4k-3)\Rightarrow 88k^2-170k+57=0$
	$88k^2-170k+57=0\Rightarrow k=\ldots$
	$k=\frac{3}{2}$