Medium Questions

1 (a) $p(x) = 15x^3 + 22x^2 - 15x + 2$

Find the remainder when $p(x)$ is divided by $x + 1$.

(2 marks)

(b) (i) Show that $x + 2$ is a factor of $p(x)$.

[1]

(ii) Write $p(x)$ as a product of linear factors.

[3]

(4 marks)

Hard Questions

1 (a) The polynomial $p(x) = 6x^3 + ax^2 + bx + 2$, where a and b are integers, has a factor of $x - 2$.
Given that $p(1) = -2p(0)$, find the values of a and b.

(4 marks)

(b) Using your values of a and b,

(i) find the remainder when $p(x)$ is divided by $2x - 1$

(ii) factorise $p(x)$.

(4 marks)

2 (a) $p(x) = ax^3 + 3x^2 + bx - 12$ has a factor of $2x + 1$. When $p(x)$ is divided by $x - 3$ the remainder is 105.

Find the value of a and of b.

(5 marks)

(b) Using your values of a and b, write $p(x)$ as a product of $2x + 1$ and a quadratic factor.

(2 marks)

(c) Hence solve $p(x) = 0$.

(2 marks)

3 (a) $\mathrm{p}(x) = 6x^3 + ax^2 + 12x + b$, where a and b are integers.

$\mathrm{p}(x)$ has a remainder of 11 when divided by $x - 3$ and a remainder of -21 when divided by $x + 1$.
Given that $\mathrm{p}(x) = (x - 2)Q(x)$, find $Q(x)$, a quadratic factor with numerical coefficients.

(6 marks)

(b) Hence solve $\mathrm{p}(x) = 0$.

(2 marks)

4 The three roots of $\mathrm{p}(x) = 0$, where $\mathrm{p}(x) = 2x^3 + ax^2 + bx + c$ are $x = \frac{1}{2}$, $x = n$ and $x = -n$, where a, b, c and n are integers. The y-intercept of the graph of $y = \mathrm{p}(x)$ is 4. Find $\mathrm{p}(x)$, simplifying your coefficients.

(5 marks)

Very Hard Questions

1 (a) The polynomial $\mathrm{p}(x) = ax^3 - 9x^2 + bx - 6$, where a and b are constants, has a factor of $x - 2$. The polynomial has a remainder of 66 when divided by $x - 3$. Find the value of a and of b.

(4 marks)

(b) Using your values of a and b, show that $\mathrm{p}(x) = (x - 2)\mathrm{q}(x)$, where $\mathrm{q}(x)$ is a quadratic factor to be found.

(2 marks)

(c) Hence show that the equation $\mathrm{p}(x) = 0$ has only one real solution.

(2 marks)

2 (a) $\mathrm{p}(x) = 2x^3 - 3x^2 - 23x + 12$

Find the value of $\mathrm{p}\left(\frac{1}{2}\right)$.

(1 mark)

(b) Write $\mathrm{p}(x)$ as the product of three linear factors and hence solve $\mathrm{p}(x) = 0$.

(5 marks)

3 (a) The polynomial $\mathrm{p}(x) = ax^3 + bx^2 - 19x + 4$, where a and b are constants, has a factor $x + 4$ and is such that $2\mathrm{p}(1) = 5\mathrm{p}(0)$.

Show that $\mathrm{p}(x) = (x + 4)(Ax^2 + Bx + C)$, where A, B and C are integers to be found.

(6 marks)

(b) Hence factorise $\mathrm{p}(x)$.

(1 mark)

(c) Find the remainder when $\mathrm{p}'(x)$ is divided by x.

(1 mark)